Normal Modes of Vibration of Triangular Cantilever Plates by the Ritz Method

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BLAINE WRIGHT ANDERSEN

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NORMAL MODES OF VIBRATION OF TRIANGULAR CANTILEVER PLATES BY THE RITZ METHOD

The purpose of this thesis was to find and illustrate a feasible method of studying theoretically the vibration characteristics of triangular cantilever plates which approximate delta wings or tapered fins. The investigation has been limited to plates of uniform thickness.

Using the method published by Ritz in 1909, natural frequencies and corresponding node lines have been determined for two symmetric and two antisymmetric modes of vibration of isosceles triangular plates clamped at the base and having length to base ratios of 1, 2, 4 and 7 and for the two lowest modes of right triangular plates clamped along one leg and having ratios of the length of the free leg to that of the clamped one of 2, 4 and 7. A nonorthogonal coordinate system was used which gave constant limits of integration over the area of the triangle. The coordinate transformation made it necessary to modify the functions used by Ritz in approximating deflections and to consider cross products in the integration. The integration was done numerically using tables compiled by Young and Felgar in 1949.

To check the accuracy of results, a solution was obtained to the problem of a vibrating cantilever beam of uniform depth and triangular plan view. The results obtained were found to be consistent with those obtained for the plates by using an eight term series to approximate the deflections of the symmetric plates (isosceles triangles) and a six term series to approximate the deflections of the unsymmetric plates (right triangles).

In general it was found that the frequencies of the plates were lower than those for the corresponding beams. For the symmetric plates the difference varied from 0.15% lower for the shortest plate considered to 1.3% lower for the longest. For the unsymmetrical plates it varied from 17.8% lower for the shortest to 3.7% lower for the longest. These figures show the marked affect of rotation upon the frequencies of short unsymmetric plates.

VITA

Blaine Wright Andersen was born on December 29, 1925, in Salt Lake City, Utah. After graduating from high school in 1943, he entered the University of Utah, where he completed two quarters before being called into the military service. After being discharged in 1946, he re-entered the University of Utah and received the degree of Bachelor of Science, with honors, in 1949. He then entered the graduate college of the University of Illinois where he received the degree of Master of Science in 1951. He held a teaching assistanceship during the academic years 1949-50 and 1950-51 and an Atomic Energy Commission Pre-Doctoral Fellowship during the calendar year beginning September 1, 1951. He is a member of Tau Beta Pi and Pi Tau Sigma Fraternities and an associate member of the Society of Sigma Xi.